

ON DETERMINATION OF THE MINIMUM GENUS OF  
COMPACT RIEMANN SURFACES ADMITTING A CLASS  
OF METACYCLIC AUTOMORPHISM GROUPS

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**Abstract:** Every finite group can be represented as a group of automorphisms of a compact Riemann surface of genus  $g \geq 2$ . It is of interest to determine the minimum genus of the Riemann surface on which a given finite group acts as a group of automorphisms.

**Keywords and Phrases:** Riemann surface, Fuchsian group, automorphism, epimorphism, smooth quotient, metacyclic group, genus

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## 1. Introduction

In this paper we find the minimum genus of the surface of which the group  $G = D_m \times C_p$ , where  $m$  is an odd integer greater than one and  $p$  is any odd prime, is a group of automorphisms. The minimum value of the genus  $g$  of a compact Riemann surface having  $G$ , as its group of automorphisms is obtained as

$$(i) \quad g = 1 - m \frac{p+1}{2} + m \left( p - \frac{p}{pq_1} \right), \text{ if } (p, m) = 1;$$

$$(ii) \quad g = 1 - m \frac{p+1}{2} + m \left( p - \frac{p}{q_1} \right), \text{ if } (p, m) \neq 1;$$

$$(iii) \quad g = 4, \text{ if } p = m = 3;$$

where  $m$  has the prime decomposition

$$m = q_1^{r_1} q_2^{r_2} q_3^{r_3} \cdots q_l^{r_l}; \quad r_i > 0, \quad q_1 < q_2 < \cdots < q_l, \quad q_i \geq 3.$$

Groups of automorphisms of compact Riemann surfaces constituted a glamorous topic of research during the last decade of the 20th century and for its